Math 3450 - Homework # 1 - Part A - Solutions

Part 1 - Set builder notation

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1. Find all the elements from the set $\{n \in \mathbb{Z} \mid 1 \leq n^2 \leq 100\}$.

   Solution: $-10, -9, 8, -7, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

2. Let $S = \{1, 5, 7\}$ and $T = \{-1, 0, 10, 5\}$. Find all the elements in the set $X = \{a + b \mid a \in S, b \in T\}$.

   Solution:

$$\begin{aligned} X &= \{1 + (-1), 1 + 0, 1 + 10, 1 + 5, \\ &\quad 5 + (-1), 5 + 0, 5 + 10, 5 + 5, \\ &\quad 7 + (-1), 7 + 0, 7 + 10, 7 + 5\} \\ &= \{0, 1, 11, 6, 4, 5, 15, 10, 6, 7, 17, 12\} \\ &= \{0, 1, 4, 5, 6, 7, 10, 11, 12, 15, 17\} \end{aligned}$$

3. Let $S = \{1, 5, 7\}$. Find all the elements in the set $Y = \{a^2 \mid a \in S\}$.

   Solution: $Y = \{1^2, 5^2, 7^2\} = \{1, 25, 49\}$

4. List all of the elements from $S = \{3k^2 + 1 \mid k \in \mathbb{Z} \text{ and } -1 \leq k < 4\}$

   Solution:

   $S = \{3(-1)^2+1, 3(0)^2+1, 3(1)^2+1, 3(2)^2+1, 3(3)^2+1\} = \{4, 1, 4, 13, 28\} = \{1, 4, 13, 28\}$

5. List 5 elements from the set $S = \{2x - 3y \mid x, y \in \mathbb{Z}\}$.

   Solution: 5, 2, -1, -7, and -2 are all elements in S. This is because $5 = 2(1) - 3(1)$, $2 = 2(1) - 3(0)$, $-1 = 2(7) - 3(5)$, $-7 = 2(-2) - 3(1)$, and $-2 = 2(2) - 3(2)$.

6. Use set-builder notation to write the set of all positive odd numbers.

   Solution: Possible answer: $\{2k - 1 \mid k \in \mathbb{N}\}$. This works because

$$\begin{aligned} \{2k - 1 \mid k \in \mathbb{N}\} &= \{2(1) - 1, 2(2) - 1, 2(3) - 1, 2(4) - 1, \ldots\} \\ &= \{1, 3, 5, 7, \ldots\}. \end{aligned}$$

Part 2 - Basic set operations

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7. Let $A = \{1, 5, -12, 100, 1/3, \pi\}$, $B = \{5, 1, -12, 18, -1/3\}$, $C = \{10, -1, 0\}$, $D = \{1, 2\}$, and $E = \{1, -1\}$. Calculate the following:

   (a) $A \cup B$
   Solution: $\{1, 5, -12, 100, 1/3, \pi, 18, -1/3\}$

   (b) $A \cap B$
   Solution: $\{1, 5, -12\}$

   (c) $A \cap C$
   Solution: $\emptyset$

   (d) $A \cap \emptyset$
   Solution: $\emptyset$

   (e) $B \cup \emptyset$
   Solution: B

   (f) $D \times E$
   Solution: $\{(1, 1), (1, -1), (2, 1), (2, -1)\}$

   (g) $(D \cap A) \times (E \cup D)$
   Solution: $D \cap A = \{1\}$, $E \cup D = \{1, 2, -1\}$, $(D \cap A) \times (E \cup D) = \{(1, 1), (1, 2), (1, -1)\}$

   (h) $C \times D$
   Solution: $\{(10, 1), (-1, 1), (0, 1), (10, 2), (-1, 2), (0, 2)\}$

   (i) $A - B$
   Solution: $\{100, 1/3, \pi\}$

   (j) $C - A$
   Solution: C

   (k) $A - \emptyset$
   Solution: A

   (l) $A \cup B \cup C \cup D$
   Solution: $\{-12, -10, -1, -1/3, 0, 1/3, 1, 2, \pi, 5, 18, 100\}$

   (m) $A \cap B \cap D$
   Solution: $\{1\}$

   (n) $A \cap B \cap C$
   Solution: $\emptyset$

8. Let $A = \{1\}$. List the elements of the power set $\mathcal{P}(A)$.

   Solution:

   $\mathcal{P}(A) = \{\emptyset, \{1\}\}$

9. Let $B = \{-1, 3\}$. List the elements of the power set $\mathcal{P}(B)$.

   $\mathcal{P}(B) = \{\emptyset, \{-1\}, \{3\}, \{-1, 3\}\}$

10. Let $C = \{2, 4, 6\}$. List the elements of the power set $\mathcal{P}(C)$.

    $\mathcal{P}(C) = \{\emptyset, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}\}$

Part 3 - Families of sets

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11. Let $A_n = \{x \in \mathbb{Z} \mid -n \leq x \leq n\}$.

    (a) List the elements in the sets A_1, A_2, A_3, and A_4.

    Solution:

    $A_1 = \{-1, 0, 1\}$ $A_2 = \{-2, -1, 0, 1, 2\}$
    $A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$
    $A_4 = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

    (b) Calculate $\bigcap_{n=2}^{\infty} A_n$ and $\bigcup_{n=5}^{\infty} A_n$.

    Solution:

    $\bigcap_{i=2}^{\infty} A_n = \{-2, -1, 0, 1, 2\}$
    $\bigcup_{i=5}^{\infty} A_n = \mathbb{Z}$

12. Let $A_n = \{-2n, 0, 2n\}$.

    (a) List the elements in the sets A_1, A_2, A_3, and A_4.

    Solution:

    $A_1 = \{-2, 0, 2\}$
    $A_2 = \{-4, 0, 4\}$
    $A_3 = \{-6, 0, 6\}$
    $A_4 = \{-8, 0, 8\}$

    (b) Calculate $\displaystyle\bigcap_{n \in \mathbb{N}} A_n$ and $\displaystyle\bigcup_{n \in \mathbb{N}} A_n$.

    Solution:

    $\bigcap_{n \in \mathbb{N}} A_n = \{0\}$
    $\bigcup_{n \in \mathbb{N}} A_n = \{2k \mid k \in \mathbb{Z}\}$, ie the set of even integers